Eliminating adjacent-channel interference

by P. L. Taylor, M.A., F.I.E.E., F.I.M.A., *University of Salford*

Adjacent-channel interference between amplitude-modulated signals can be overcome, even when the carrier frequencies are so close together that the sideband of one signal overlaps the carrier of the other.

The problem of adjacent-channel interference has been with us almost since radio communiction began. Fig. 1 illustrates the situation in which it arises: the carrier frequency of an unwanted amplitude-modulated signal *U* is too close to the carrier frequency of a wanted signal *W*. The result is that some of one sideband of *U* intrudes into the part of the spectrum occupied by *W*. A receiver tuned to *W* must have a pass-band sufficiently wide to accept the sidebands of *W*, and so cannot reject the unwanted sideband of *U*. The audible result, after detection, is unintelligible and annoying "sideband. splash" or "monkey chatter" caused by the beating of the unwanted frequencies with the carrier of *W*.

If *U* is not too close to *W*, as in Fig. 1(a), then it is possible to design the receiver to accept only the "clean" sideband of *W* (which contains all the modulation information in itself) and to treat the result as a single-sideband signal; but this requires very sharp and precise filtering, which of course is expensive. If the two carrier frequencies are as close together as is shown in Fig. 1(b) it has been generally thought that there is nothing one can do about the situation. In addition to the monkey chatter one must put up with an inter-carrier whistle at the difference frequency between the two carriers.

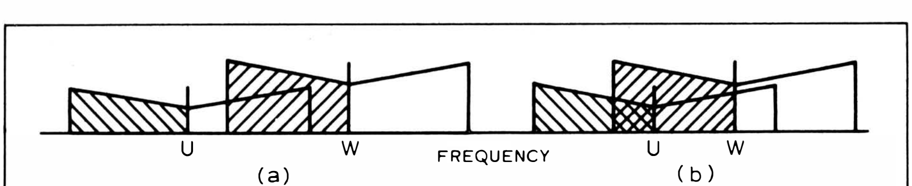


Fig. 1. If the carrier of an unwanted signal U *is too close to that of a wanted signal* W *there is interference. In case (a) it is possible to filter out the "clean" sideband of* W. *Up to now it has been thought that nothing could be done in a situation such as (b).*

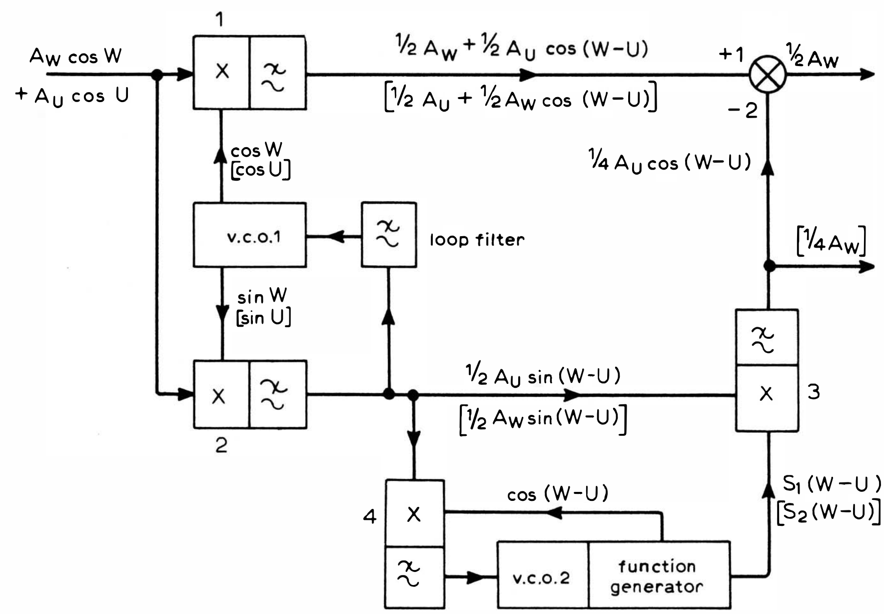


Fig. 2. Block diagram for both methods of overcoming interference.

Here are two methods[1,2] which provide solutions to the problem. Both begin with synchronous demodulation of the wanted signal, as in the homodyne and synchrodyne receivers.† For brevity, the wanted signal will be represented by A_W cos *W*, where $W=2\pi f_W t$, f_W being the frequency of the wanted carrier. Similarly, the unwanted signal will be represented by A_U cos *U*. We want to recover A_W, uncontaminated by A_U.

In synchronous demodulation, the wanted carrier is multiplied by an oscillation having exactly the same frequency and phase. The result is

$$A_W \cos W \times \cos W = \tfrac{1}{2}A_W + \tfrac{1}{2}A_W \cos 2W$$

(Table I may be a helpful reminder).

Thus the wanted signal A_W is recovered, together with an oscillation at twice the carrier frequency, which is easily removed by filtering.

Table I

$\cos A \cos B = \tfrac{1}{2}\cos(A-B) + \tfrac{1}{2}\cos(A+B)$
$\sin A \sin B = \tfrac{1}{2}\cos(A-B) - \tfrac{1}{2}\cos(A+B)$
$\sin A \cos B = -\tfrac{1}{2}\sin(A-B) + \tfrac{1}{2}\sin(A+B)$

$\cos(-C) = \cos C$; $\sin(-C) = -\sin C$

First method

Figure 2 is the block diagram, in which the expressions in square brackets should be ignored, since they relate to

† This is history repeating itself. When Professor Tucker did his work on the synchrodyne he was led to consider the present problem, and suggested an approximate solution. Some while ago the author was casting round for projects for his final-year undergraduate students and thought it might be interesting to see what could be made of the synchrodyne using modern technology. He, too, was led to consider the problem; this time the suggested solution is exact.

the second method. The combined signals are applied to demodulator 1, where they are multiplied by cos W. The output of this demodulator (after filtering) is now $\frac{1}{2}A_W + \frac{1}{2}A_U \cos(W-U)$. The second term in this expression is the audible interference. The multiplier cos W is obtained from a voltage-controlled oscillator VCO_1 which is phase-locked to the wanted carrier via demodulator 2. VCO_1 produces quadrature outputs. The phase-lock loop will settle itself so that the v.c.o. output which is presented to demodulator 2 is in quadrature with the wanted signal, so this output must be represented by sin W and the quadrature output will be cos W. It is arranged that when capture has occurred the loop bandwidth is reduced to about 1Hz by extra filtering so that the oscillator is not disturbed by the other frequencies present in the signals. Also, the loop includes an integrator so that the phasing is exact.

Now the output of demodulator 2 contains the component $\frac{1}{2}A_U \sin(W-U)$, but no component involving A_W. The clue is too obvious to be missed: if the phase of this oscillation could be changed from $\sin(W-U)$ to $\cos(W-U)$ it could be used to cancel the unwanted component in the output of demodulator 1. This could be done by multiplying, in a third demodulator, by $\sin 2(W-U)$:

$$\tfrac{1}{2}A_U \sin(W-U) \times \sin 2(W-U) = \tfrac{1}{4}A_U \cos(W-U) - \tfrac{1}{4}A_U \cos 3(W-U)$$

Thus the desired phase-shifting has been accomplished but at the cost of introducing a 3rd-harmonic oscillation, and, if $(W-U)$ is small, it may not be possible to filter it out. But if $\frac{1}{2}A_U \sin(W-U)$ is multiplied by the series

$$S_1(W-U) = \sin 2(W-U) + \sin 4(W-U) + \ldots + \sin 2n(W-U),$$

the result is:

$$\tfrac{1}{4}A_U \sin(W-U)\, S_1(W-U) = \tfrac{1}{4}A_U \cos(W-U) - \tfrac{1}{4}A_U \cos(2n+1)(W-U).$$

The intermediate products give rise to sum- and difference-frequency terms which cancel, leaving the interfering oscillation at a frequency which may be made as high as desired by a suitable choice of n; this oscillation may now be filtered out easily. Thus, the desired cancellation signal is obtained, and processing is completed as shown in Fig. 2.

A waveform, whose Fourier series components form $S_1(W-U)$, is obtained from a function generator which is described later. The generator is phase-locked via VCO_2 and demodulator 4 to the beat frequency $(W-U)$. Note that the series S_1 is one in which all the first $(n-1)$ harmonics are equal in amplitude to the fundamental, which has a frequency twice that of the beat frequency.

Second method

If the unwanted signal is stronger than the wanted signal it will probably be easier to lock VCO_1 on to the unwanted carrier, so that (taking the expressions in brackets in Fig. 2) the output of demodulator 2 becomes $\frac{1}{2}A_W \sin(W-U)$. Thus, the unwanted signal is rejected directly at this stage, but the problem now is that the wanted signal is modulated on a carrier frequency that lies within the audio range.

The wanted signal could be demodulated by multiplying by $\sin(W-U)$:

$$\tfrac{1}{2}A_W \sin(W-U) \times \sin(W-U) = \tfrac{1}{4}A_W - \tfrac{1}{4}A_W \cos 2(W-U)$$

but this introduces an interfering oscillation, at twice the beat frequency, which may still be too low to filter out. But if $\frac{1}{2}A_W \sin(W-U)$ is multiplied by the series

$$S_2(W-U) = \sin(W-U) + \sin 2(W-U) + \ldots + \sin(2n+1)(W-U)$$

the result is

$$\tfrac{1}{2}A_W \sin(W-U) S_2(W-U) = \tfrac{1}{4}A_W - \tfrac{1}{4}A_W \cos(2n+2)(W-U).$$

The intermediate products give rise to sum- and difference-frequency terms which cancel, leaving the interfering oscillation at a frequency which may be made as high as desired by suitable choice of n; it is thus easily filtered out. In this method the wanted signal is taken from the output of demodulator 3.

Function generation

It would be possible to generate the series S_1 or S_2 by taking a number of oscillators, of appropriate harmonic frequencies, and phase-locking them together and to the beat frequency $(W-U)$. But this would be clumsy, and would also require that the demodulator 3 should be a true multiplier. The simplicity of a switching demodulator may be retained as follows.

In normal use a switching demodulator acts to change the sign of the signal to be demodulated in step with alternate half-cycles of the multiplier oscillation. That is, it effectively multiplies the signal by a square wave switching function f, drawn as the solid line in Fig. 3, which alternates between the values +1 and −1 with the same period T as the multiplier oscillation. As drawn in Fig. 3, the function f is odd (in the mathematical sense), that is, $f(-t) = -f(t)$, and the graph has rotational symmetry about the point $t=0$. Hence its Fourier series consists of odd functions (sine terms) only:

$$f(t) = \frac{4}{\pi}\left[\frac{2\pi}{T} + \frac{1}{3}\sin 3\frac{2\pi}{T} + \frac{1}{5}\sin 5\frac{2\pi}{T} + \ldots\right]$$

Thus, the demodulator does multiply the signal by the required frequency (the first term in the series). It also multiplies by the higher frequencies in the series, but the results are usually filtered out.

Now, suppose that two extra edges are introduced, at t_1 and t_2, to give the dotted wave. Since S_1 consists only of sine terms the rotational symmetry must be preserved, by introducing corresponding edges at $-t_1$ and $-t_2$. Now t_1 and t_2 can be chosen at will; the question is, can we choose them so that the first two harmonics of the new waveform have amplitudes equal to the fundamental? The answer is yes, and the result is quite general: if n extra edges are introduced, then the first n harmonics can be made to have amplitudes equal to the fundamental.

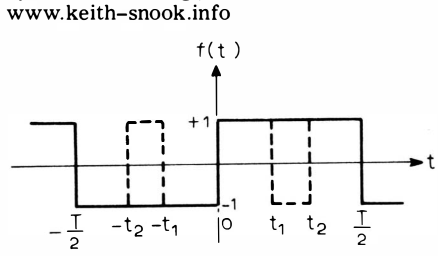
The correct instants $t_1, t_2 \ldots t_n$ are found as follows. The expression for the Fourier series of the new waveform is found in the usual way, and from it the conditions that the coefficients of the first n harmonics shall be equal are found. This results in a set of simultaneous equations in the unknown t. However, the equations are non-linear, so the solution of them is best entrusted to a computer.

Thus a square waveform can be designed such that the first terms in its Fourier series form S_1. A similar argument leads to a waveform the terms of which form S_2. There is a small complication in this case because only the odd harmonics are required. Both series continue with higher-order terms, but these do not matter because the unwanted products to which they give rise will be filtered out anyway.

The waveforms may be generated quite easily by digital techniques. VCO_2 is made a high-frequency oscillator, the cycles of which are presented to a digital counter. The counter output is presented in turn to a number of digital comparators (one for each edge) which are hard-wired with numbers defining the instants at which the edges occur. Whenever a coincidence is detected, an edge is generated by triggering a bi-stable.

In an alternative method, numbers representing the *differences* between successive edges are placed in a read-only memory (r.o.m.). A presettable counter is loaded with the first number, and is counted down to zero by VCO_2. When zero is reached an edge is generated, the number in the next address in the r.o.m. is loaded into the counter and so on until the cycle is

Fig. 3. Illustrating the derivation of the special switching functions.



completed and control is returned to the first address in the r.o.m. This method is more economical of hardware, and more flexible because the numbers for several series can be stored in one r.o.m. Any waveform can be selected simply by choosing the appropriate starting address.

Sidebands

Though the mathematical analysis given above indicates that the methods should work, and experiment shows that they do work, it is not so far clear exactly how it is that the overlapping sidebands are disentangled.

Take as an example the first method. Suppose that initially VCO_1 has not locked on to the wanted signal, but is running at some frequency F higher than W. The output of both demodulators 1 and 2 is a group of signals at the sum- and difference-frequencies, as in Fig. 4(a). Only the lower frequency group is retained; the other is eliminated by the low-pass filter.

Now suppose that F is reduced towards W. The lower frequency group moves towards zero frequency and a stage is reached when some of the sideband frequencies of the wanted signal should become negative, as shown at (i) in Fig. 4(b). The practical effect differs in the two demodulators. In the case of demodulator 1 the product is cos $W \times$ cos F, and therefore is also a cosine. The cosine of a negative quantity is the same as the cosine of the same positive quantity (see Table I) so the negative frequency components are reflected about zero frequency, without change of sign, to become positive frequency components as shown at (ii). In demodulator 2, which is multiplying cos $W \times$ sin F, the output is a sine; and the sine of a negative quantity is minus the sine of the same positive quantity, so in this case the reflected components must be shown as negative, as at (iii).

Finally, let F be reduced to equal W so that VCO_1 locks. In the output of demodulator 1 the lower sideband of the wanted signal folds back to reinforce the upper sideband, and both now start from zero frequency, i.e. the wanted signal is demodulated. This is shown in Fig. 4(c). The unwanted signal is modulated on to the beat frequency $(W-U)$ and its lower sideband is folded back. In the output of demodulator 2, Fig. 4(d), the sidebands of the wanted signal exactly *cancel* each other, being of opposite sign, so the wanted signal does not appear in the output of this demodulator.

Now consider the effect of multiplying (d) by the series S_1. The resulting spectrum of the output of demodulator 3 is shown at (e). First, there are sum- and difference-components centred on the frequency of the first term in the series, $2(W-U)$. We are now dealing with a sine×sine product, which is a cosine, so the part of the lowest sideband which is partially reflected about zero is reflected without change of sign; and the sum-frequency components have a negative sign.

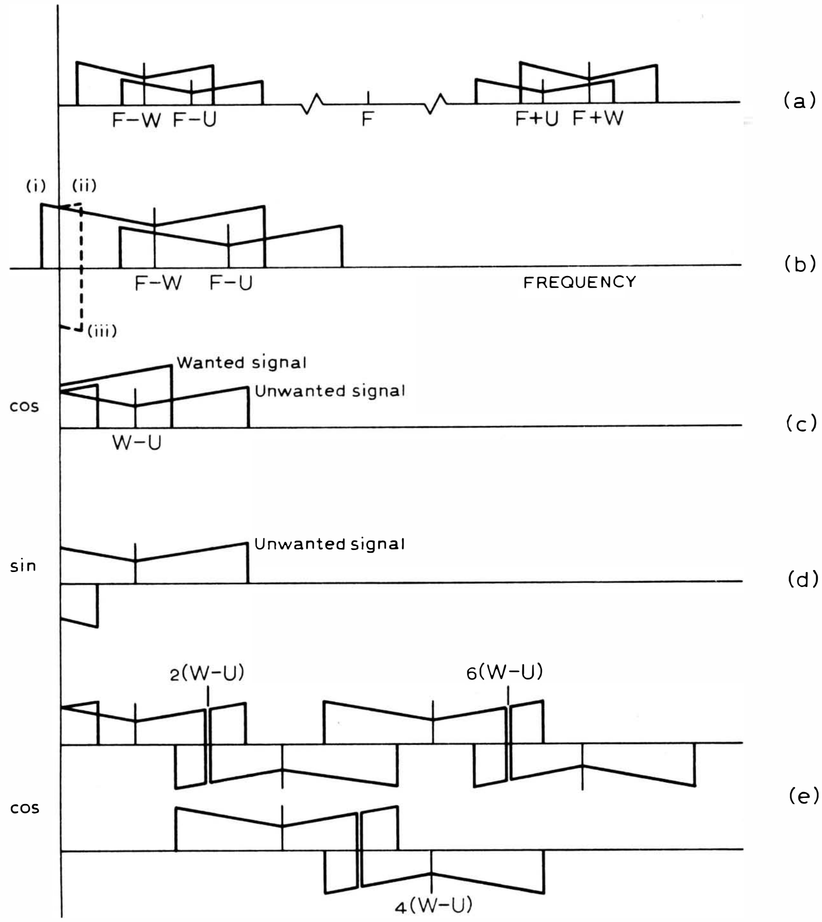


Fig. 4. (a) Result of multiplying the incoming signals by a frequency F *greater than* W. *(b) If* F *is only slightly greater than* W *some reflection of the lower sideband occurs. (c), (d) Outputs of demodulators 1 and 2 respectively when* F = W. *(e) The result of multiplying (d) by the series* S_1.

For clarity, the sum- and difference-frequency components centred on the frequency of the next term, $4(W-U)$, are shown on a lower line. The diagram is drawn for the case where it is necessary to go only as far as the third term in the series, of frequency $6(W-U)$. When all the various bands are added together there is a lot of mutual cancellation; there are left only the lowest group of frequencies, which are now of the right form for subtraction from (c), and the highest group; in between there is a big gap, so that filtering out the highest group is easy.

The foregoing description makes it clear that the methods are really exploiting the fact that an a.m. signal has two symmetrical sidebands to effect mutual cancellation of unwanted signals. It is also clear that the cancellation will be less than exact if the sidebands suffer differential gain and/or phase shift in their passage through the r.f. and i.f. stages of a receiver. It is unlikely, therefore, that these methods will form a satisfactory basis for an "add-on" unit for an existing receiver, in which these aspects of performance will probably not have received much attention. It is also clear that, unfortunately, they will not work for s.s.b.!

I am very grateful to Mr L. J. Unsworth for constructing the experimental apparatus in which these ideas were tested.

References

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Space shuttle comms

The Battelle Institute say the communications industry could save millions of dollars in the 1980s if their satellites used the space transportation system of which the shuttle is a part. A NASA funded study is being carried out at Battelle's Columbus Laboratories with five satellite manufacturers to make their systems compatible with s.t.s.