RMS watt, or not?

When you see a power amplifier advertised as 100 watts rms, what – if anything – does it mean? Lawrence Woolf explains.

I sometimes see the term 'watts rms' used in published text and advertisements. As 'rms' may be correctly applied to voltage and current it seemed worth while to examine the implications and meaning, if any, of applying it to power. This requires a degree of mathematics. Even if you are not interested in the maths, you might still find the summary interesting.

DC power

If you apply a constant dc voltage to a fixed resistor, the current through the resistor and the power dissipated in it are easily calculated using Ohm's law,

$$I = \frac{E}{R} \qquad (1)$$

where E is the applied voltage, or electromotive force, R is the resistance in ohms and I is the load current in amps. For power,

$$W = E \times I \qquad (2)$$

or,

$$W = \frac{E^2}{R} \qquad (3)$$

where W is the power dissipated in watts.

This power may be used in various ways but here we only need to consider that heat is generated. The rise in temperature that results from the power applied will depend on factors such as the power dissipated and the power radiated.

After a period of time a steady state is achieved. At this point, the radiated power balances the applied power and the load stays at a constant temperature somewhere above the ambient temperature.

As an example, a soldering iron takes some time to reach its working temperature and then should maintain it steadily.

AC power

As far as heating the soldering iron is concerned, it does not matter whether the energy applied involves an alternating voltage or a direct voltage. The next task is to define the alternating voltage that will supply the same heating energy as the direct voltage.

The problem is that the alternating voltage voltage is, by definition, constantly varying. If the iron is powered by our mains at a frequency of 50Hz then each repeated sinusoidal cycle takes 20ms which is $^1/_{50}$th of a second, **Fig. 1a)**. At time 0, the voltage is zero but rising.

After 5ms, the voltage reaches its positive peak, which I will call E_P. After a further 5ms, at 10ms, the voltage is back to zero but falling. At 15ms the voltage reaches its negative peak, $-E_P$. At 20ms the voltage is back to zero again and rising again as the sequence is repeated.

During this cycle the voltage has reached a positive peak and a negative peak. It has also been zero three times and has passed through every possible intermediate value twice. Which of these values, if any, could be used as a definitive value?

What is needed is a value that is numerically the same as for the direct voltage that will heat the iron to the same temperature. This is clearly not the peak value as, for most of the time, the magnitude of the voltage is below this.

We need to find a constant that we can multiply the peak value by to give the equivalent heating power of a known dc voltage. The constant seems likely to be less than one. This now raises the problem of also defining the alternating power which is also varying during the cycle.

In Fig. **1b)** the ac voltage waveform is shown together with the power waveform. As the power is proportional to E^2 both the positive and negative voltage peaks correspond to positive power peaks. When the voltage is zero, so is the power.

The resulting waveform is a raised cosine and has a frequency that is twice that of the voltage waveform. The load is assumed to be purely resistive so the power cannot, at any time, have a negative value.

A mathematical description is given by saying that,

$$E_t = E_P \sin(\omega t) \qquad (4)$$

where E_t is the instantaneous voltage at time t, E_P is the peak voltage and ω is the angular frequency in radians per second, i.e. $2\pi f$ where, f is the frequency in hertz.

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By using equations 3 and 4, with appropriate subscripts, the power waveform may be defined,

$$W_t = E_p^2 \times \frac{\sin^2(\omega t)}{R} \quad (5)$$

$$= E_p^2 \times \frac{1-\cos(2\omega t)}{2R} \quad (6)$$

where W_t is the instantaneous power at time t and R is now the resistance of the soldering iron.

Equation 6 shows, by using a standard trigonometric substitution, that the power waveform is indeed a raised cosine at twice the frequency of the voltage waveform.

Figure **1c)** shows that the average value of the power waveform is half the peak value. The waveform is symmetrical about the average power line,

$$W_{AV} = \frac{W_P}{2} \quad (7)$$

where W_{AV} is the average power and W_P is the peak power.

This average power is the heating power provided to the soldering iron and must be equivalent to the original dc power if it causes the iron to operate at the same temperature.

It is now possible to equate the dc and ac powers to derive the required constant to equate equivalent dc and ac voltages. Using equation 3, but with the peak ac values,

$$W_P = \frac{E_P^2}{R} \quad (8)$$

so that,

$$W_{AV} = \frac{E_P^2}{2R} \quad (9)$$

but,

$$W_{AV} = W_{dc} = \frac{E_{EQ}^2}{R} \quad (10)$$

where W_{dc} is the power from the dc source and E_{EQ} is the equivalent direct voltage.

We can now use equations 9 and 10 to find the relationship between the peak alternating voltage, E_P, and the equivalent direct voltage, E_{EQ}.

$$\frac{E_P^2}{2R} = \frac{E_{EQ}^2}{R} \quad (11)$$

This re-arranges to,

$$E_{EQ}^2 = \frac{E_P^2}{2} \quad \text{so that,} \quad E_{EQ} = \frac{E_P}{\sqrt{2}}$$

or

$$E_{EQ} \approx E_P \times 0.707$$

We now have our conversion factor that gives us the equivalent direct voltage that will produce the same heating effect as an alternating voltage, of known peak value, in a constant resistive load. This equivalent voltage is more commonly known as the rms voltage or E_{RMS} which now needs to be defined.

Root-mean-square

I have now stated that when an alternating voltage is applied to a resistive load it will have the same heating effect, in that load, as a direct voltage whose value is numerically the same as the rms value of the alternating voltage. I will next explain rms, and how to calculate it.

RMS is the abbreviation for root-mean-square. It is used in statistics as well as physics so is a useful concept. In order to apply it to a given waveform, such as a sine wave, rms can be considered in stages.

- Divide the waveform into narrow vertical slices, one is shown in Fig **1d)**. Each slice is narrow enough to consider it as having a single amplitude value.
- Square each value.
- Sum the values then divide the sum by the number of slices. You now have the mean of the squares.
- Finally, take the square root. This gives the square root of the mean of the squares of the sliced waveform.

The equation used in statistics is,

$$RMS\ value = \sqrt{\frac{(x_1^2 + x_2^2 + x_3^2 ... x_n^2)}{n}} \quad (12)$$

where x is the size of each slice or sample and n is the number of slices.

As we are considering a repetitive waveform that is easily defined mathematically there is a simpler way of performing the calculation. At least it is simpler for those of us who are familiar with integral calculus. The appropriate form of the equation is given by,

$$E_{RMS} = E_P \sqrt{\frac{1}{T}\int_0^T \sin^2(\omega t)dt} \quad (13)$$

where T is the time period under consideration.

The time period could be any that defines the symmetry of the waveform. For a sine wave just a quarter cycle is adequate as the following quarter cycles may be shown to be rotations or reflections of the first one. Therefore a suitable value for T is $\pi/2$, although the same result is achieved using π or 2π.

In order to solve equation 13 we can put in this value and use the same trigonometric substitution used in equation 6.

$$E_{RMS} = E_P \sqrt{\frac{2}{\pi}\int_0^{\frac{\pi}{2}} \frac{1}{2}(1-\cos(2\omega t))dt} \quad (14)$$

$$= E_P \sqrt{\frac{2}{\pi}\left[\frac{1}{2}\left(t - \frac{1}{2}\sin(2\omega t)\right)\right]_0^{\frac{\pi}{2}}}$$

$$= E_P \sqrt{\frac{1}{\pi}\left[\left(\frac{\pi}{2} - \frac{1}{2}\sin(\omega\pi)\right) - \left(0 - \frac{1}{2}\sin(0)\right)\right]}$$

$$= E_P \sqrt{\frac{1}{\pi} \times \frac{\pi}{2}} = \frac{E_P}{\sqrt{2}} \approx E_P \times 0.707$$

This is the same result as found in equation 11. Previously it was found by considering the symmetry of a sine wave. This result has been derived using the definition of rms and may be applied to any waveform.

A true-rms voltmeter displays the rms value even if the waveform is not a sine wave. However most ac voltmeters actually measure the peak value and are scaled to divide by $\sqrt{2}$ even if the waveform is not a sine wave. Exactly the same argument applies to defining rms current as voltage.

But rms *power*?

Suppose we now apply the same calculation to the power function as we have done to the voltage function. We have found the ratio of E_P to E_{RMS} so we should be able to find the ratio of W_P to W_{RMS}.

Where we had to integrate a function involving $\sin^2(\omega t)$, we now have to integrate a function involving $\sin^4(\omega t)$. This is a little more complicated but there are standard trigonometric substitutions available that make the expression easier to handle.

Start by assuming that there is a meaningful relationship between W_{RMS} and W_P. From equations 5 and 8, $W_t=E_P^2\sin^2(\omega t)/R=W_P\sin^2(\omega t)$. This leads to the assumption that,

$$W_{RMS} = W_P\sqrt{\frac{1}{T}\int_0^T \sin^4(\omega t)dt} \qquad (15)$$

Again I am taking T as $\pi/2$. Using the same substitution as in equation 6,

$$\sin^2(\omega t) = \frac{1}{2}(1-\cos(2\omega t))$$

therefore,

$$\sin^4(\omega t) = \frac{1}{4}(1-2\cos(2\omega t)+\cos^2(2\omega t))$$

also,

$$\cos^2(2\omega t) = \frac{1}{2}(1+\cos(4\omega t))$$

so that,

$$\sin^4(\omega t) = \frac{1}{4}\left(1-2\cos(2\omega t)+\frac{1}{2}(1+\cos(4\omega t))\right)$$

$$=\frac{3}{8}-\frac{1}{2}\cos(2\omega t)+\frac{1}{8}\cos(4\omega t)$$

Substituting in equation 15 now gives,

$$W_{RMS} = W_P\sqrt{\frac{2}{\pi}\int_0^{\frac{\pi}{2}}\left(\frac{3}{8}-\frac{1}{2}\cos(2\omega t)+\frac{1}{8}\cos(4\omega t)\right)dt}$$

$$= W_P\sqrt{\frac{2}{\pi}\left[\frac{3t}{8}-\frac{1}{4}\sin(2\omega t)+\frac{1}{32}\sin(4\omega t)\right]_0^{\frac{\pi}{2}}}$$

$$= W_P\sqrt{\frac{2}{\pi}\left(\frac{3\pi}{16}\right)} = W_P\sqrt{\frac{3}{8}} \approx W_P \times 0.612$$

but from equation 7, $W_{AV}=W_P/2$ or $W_P=2W_{AV}$. This means that $W_{RMS}\approx W_{AV}\times 1.225$.

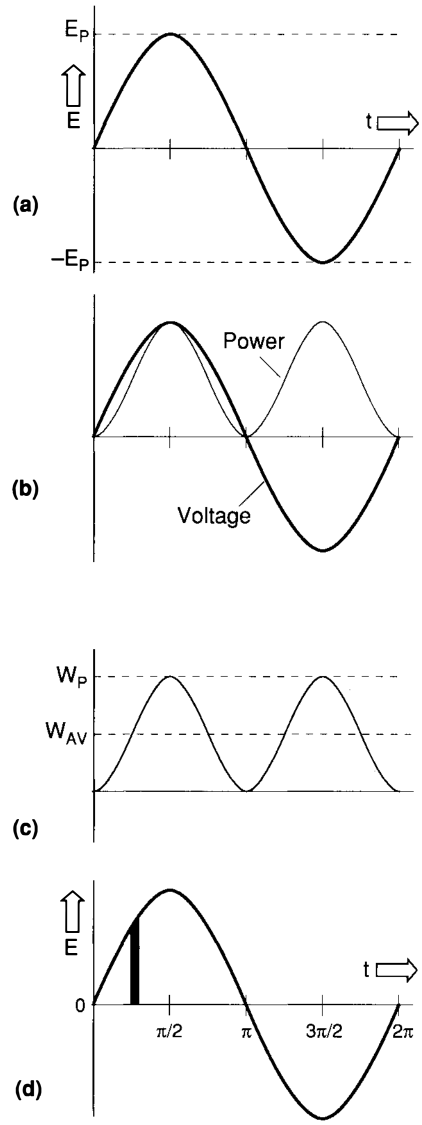

Fig. 1. 50Hz sine wave, a), voltage and power waveforms, b), power waveforms showing peak and average, c) and a sine wave with narrow slice, d).

In summary

The implication of all this is that a transmitter that puts out 100W average power might also be said to have an output of 122.5W rms. This is hardly the same thing and would seem to have no practical or physical significance. It is merely a mathematical curiosity.

Alternatively one might assume that if someone claims an output of 100W rms then they are actually transmitting 81.63W average. The fact that it can be calculated does not, in itself, imply that it could be useful.

The only useful result is that the product of rms voltage and rms current is average power. It is not rms power – even if it looks like a logical expectation. This is the same for mains frequency power, audio power and radio frequency power.

I suspect that those that use the term probably mean 'average power under continuous sine wave'. In this case a term such as 'continuous average power' would seem more appropriate, especially if an unregulated power supply is used.

If anyone knows of a genuine reason for using the term 'watts rms' then please let me know (*lawrence@itl.net*). ■