The Design and Use of Moving-coil Loudspeaker Units

A survey of facts and current theories

by E. J. Jordan

What is the aim of a loudspeaker? "To reproduce the electrical input signal as accurately as possible"! Try again. "To reproduce the original sound as realistically as possible"! The first is an objective definition, the second is subjective and much more appropriate for the following reason. No loudspeaker is perfect and distortion of the following kinds will always occur to some degree—frequency, transient, harmonic intermodulation, and phase.

Now it is often possible for the loudspeaker engineer to trade an increase in one kind of distortion for a reduction in another. How does he determine a balance? To add to the confusion the ear is much more sensitive to some kinds of distortion than others, and sensitivity varies with the individual, so we are back to the second subjective definition. But again this has its drawbacks. Some loudspeakers can achieve a breathtaking reality but only with certain inputs and in particular environments. They have what I would call *prima donna* temperaments. On the other hand, many modern loudspeakers rarely allow the listener to escape from the fact that the sound is "canned" but most of the time they are more than just acceptable and rarely intolerable. (Most of the monitor loudspeakers I have heard fit this category.) These two extremes are quoted to further illustrate the problem of defining the aim of a loudspeaker and until this is done, we cannot begin to discuss the design.—"The aim of a loudspeaker is to make money"! Now we're getting there. One may regret that loudspeaker manufacturers are not altruistic missionaries, but getting things into their right perspective we can now state "The aim of a loudspeaker is to provide a standard of quality judged by the widest possible market as providing the highest degree of realism, when fed from the signal sources available, consistent with economic viability." This means that the greatest number of people get the best value for money—so there is a measure of altruism after all.

To meet the above criteria a loudspeaker must always have its distortions in *balance*. The more expensive a loudspeaker the lower should be the various types of distortion—*but still in balance.*
costing the earth and sporting a very wide bandwidth will be most unacceptable if there is not for example an appropriate reduction

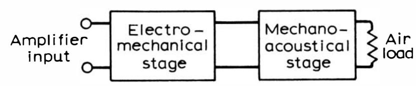

Fig. 1. The loudspeaker as a two-stage energy converter.

in transient and intermodulation distortion.

How then can we design to meet criteria that are so subjective? The road to loudspeaker design starts off with precise mathematical analysis: further along we have to rely on well established theory which itself reduces speculation, and finally we have the engineers "feel" for the subject—pure artistry!

Although we may have the most advanced equipment to help us on the way, in the end we must make the final analysis with the help only of a pair of experienced ears coupled to an open mind.

Objective analysis

The loudspeaker may be regarded as a two-stage energy converter. It converts electrical energy to mechanical energy, and this to acoustical energy as depicted in Fig. 1.

The overall conversion must be effected with the maximum efficiency and minimum distortion. (Distortion is used here in the general sense).

One prerequisite would appear to be to match the load impedance to that of the generator. In practice this can only be achieved over a restricted frequency range but is nevertheless very relevant.

Opening up the boxes in Fig. 1 we have the circuits in Figs 2(a) and (b). Circuit (a) is how the system appears from the point of view of the air load on the cone and (b) shows it as seen by the amplifier. In both cases mechanical and acoustical components are represented by electrical symbols.

Radiation impedance

For the purpose of this article the loudspeaker is assumed to be on an infinite baffle. The air load appears in Fig. 2(a) as a mechanical impedance on the cone surfaces and is represented by the radiation resistance R_{MA} and the radiation mass L_{MA} in series. Unlike true electrical components, however, both these components vary strangely with frequency. This is shown in Fig. 3 and full expressions for these are developed in Appendix 1. It is also shown that the sound distribution pattern changes, becoming more directional at high frequencies.

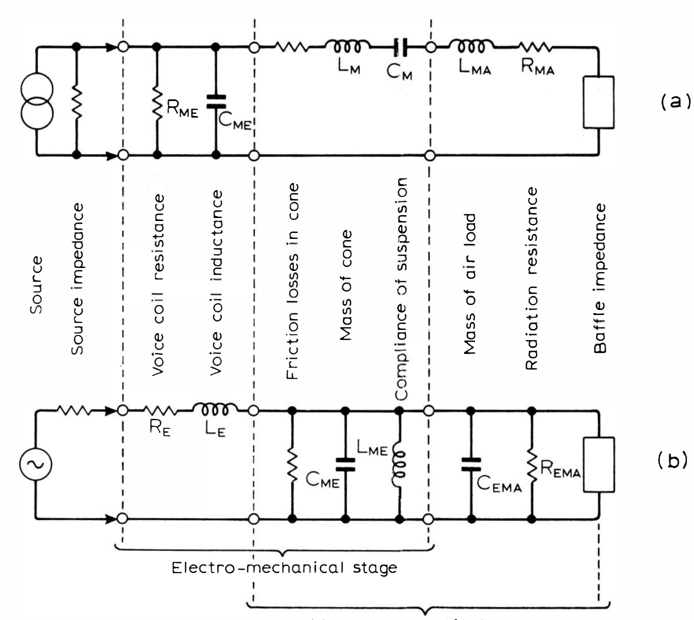

Fig. 2. The effective speaker circuit as seen (a) by the air and (b) by the amplifier.

In the case of moving-coil systems the radiation mass may be neglected since it appears in series with and is very much less than the mechanical mass of the cone L_{MA}.

The radiation resistance R_{MA} is the component in which we actually develop the sound power P_{MA}. This is given by the mechanical equivalent of Ohm's law

$$P_{MA} = v^2 R_{MA}$$

where v is the velocity of motion. From Appendix 1 we see that the value of R_{MA} is determined by the dimensions of the cone, the frequency, and a constant due to the air. The frequency at which the knee in the curve occurs is determined by the cone diameter. Fig. 4 shows normalized curves for 12-in., 8-in. and 4-in. diameter cones.

For arithmetic convenience the sloping part of the curve and the horizontal part are treated separately and have their own approximate equations. From the appendix it is seen that over the sloping part R_{MA} is proportional to f^2 and the horizontal part is independent of f.

Mechanical impedance of cone assembly

The components of the impedance are shown in Fig. 2(a) and comprise the cone mass L_M, the suspension compliance C_M and some frictional losses R_M. The most significant resistive component however is usually due to the voice-coil resistance R_E in series with its inductance L_E and the amplifier output resistance (which is negligible). From the derivations in Appendix 2 these series electrical components appear as parallel mechanical components R_{ME} and L_{ME} connected via the transducing element in series with the remaining mechanical components. The lower the actual electrical resistance the higher will be the effective mechanical resistance corresponding to it.

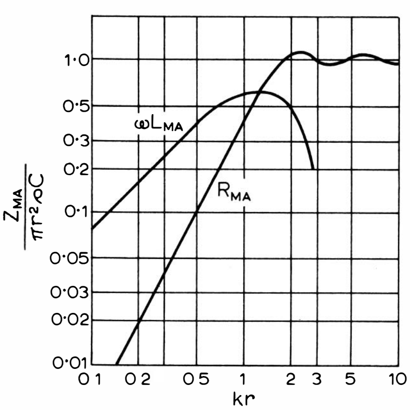

Fig. 3. Mechanical impedance of the air load on a piston surface in an infinite baffle.

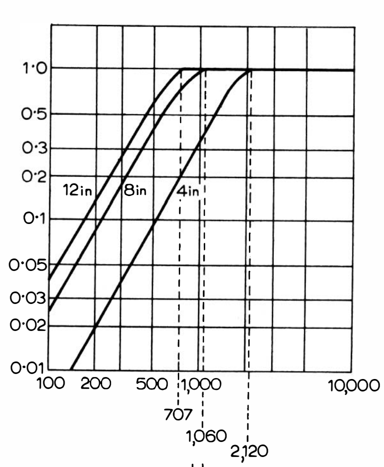

Fig. 4. Normalized R_{MA} curves for cones of 12, 8 and 4 in. diameter.

Effect of mechanical impedance on radiated power

In general the overall mechanical impedance of the cone is very much higher than that of the air load so the velocity corresponding to the current in Fig. 2(a) will be determined almost entirely by the cone. We will examine the effects of each of the cone components in turn, assuming for the moment the cone is perfectly rigid. Consider first the cone mass L_M. The velocity v is given by

$$v = \frac{F}{2\pi f L_M},$$

where F is the applied force.

Therefore radiated power is

$$P_{MA} = \frac{F^2}{4\pi^2 f^2 L_M{}^2} \cdot R_{MA}$$

Over the sloping part $R_{MA} \propto f^2$

$$\therefore\ P_{MA} \propto \frac{1}{f^2} \cdot f^2$$

i.e. P_{MA} is independent of frequency.

Over the horizontal part R_{MA} is constant with frequency.

$$\therefore\ P_{MA} \propto \frac{1}{f^2} \cdot \text{const}$$

i.e. P_{MA} falls at the rate of 12 dB/octave.

This is shown in Fig. 5(a) and is known as the condition of mass control. Due to directivity effects the axial pressure response may tend to remain constant or even rise but this will be accompanied by a greater rate of fall off axis.

With very high damping factors the resistance R_{ME} may tend to be in control. In this case:

$$P_{MA} = \frac{F^2}{R_{ME}{}^2} \cdot R_{MA}$$

Over the sloping part of R_{MA}

$$P_{MA} \propto \text{const} . f^2$$

i.e. P_{MA} rises at 12 dB/octave.

Over the horizontal part of R_{MA}

$$P_{MA} \propto \text{const} . \text{const}$$

i.e. P_{MA} is independent of frequency.

This is shown in Fig. 5(b) and is known as the condition of constant velocity.

By similar reasoning if the suspension stiffness were in control.

$$P_{MA} = f^2 4\pi^2 f^2 C_M{}^2 . R_{MA}$$

Over the sloping part of R_{MA}

$$P_{MA} \propto f^2 . f^2$$

i.e. P_{MA} rises at 24 dB/octave.

Over the horizontal part of R_{MA}

$$P_{MA} \propto f^2 \text{ const}$$

i.e. P_{MA} rises at 12 dB/octave.

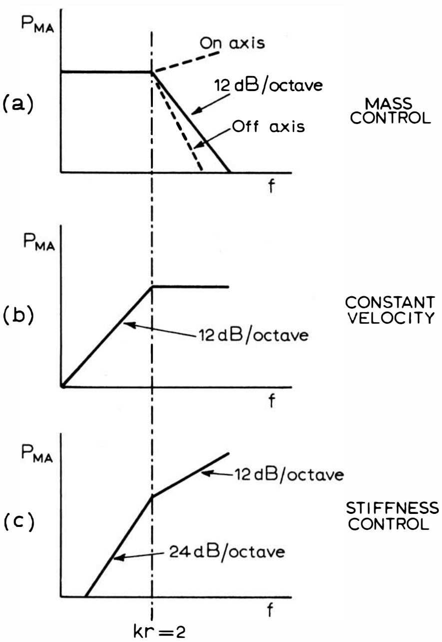

Fig. 5. Effect of mechanical impedance on radiated power assuming a rigid piston in an infinite baffle.

This is the condition of stiffness control and is represented in Fig. 5(c). This is a situation not normally encountered.

Observations (1). From this part of the work it is seen that in order to maintain a constant radiated power over the entire audio frequency range we may:

(a) Have mass control below the knee and constant velocity above it.

(b) Utilize the natural tendency for a practical cone to reduce its effective diameter as frequency rises.

(c) Use crossover techniques to bring into operation progressively smaller loudspeaker units as frequency rises.

In order to achieve (a) the cone would have to be infinitely rigid which is impossible. Method (b) relies on the fact that the cone is not infinitely rigid, and is therefore practicable. (c) is of course practicable. So we have two practicable solutions which we will discuss in detail later.

The transducing element

This is the part of the system which actually converts the electrical energy into mechanical and comprises the magnet and the voice coil. In one sense it behaves like a transformer having a turns ratio of $Bl:l$, where B is the magnetic flux density and l is the length of wire in the magnetic gap. Its other characteristic is that it inverts impedances. For example the mechanical damping resistance R_{ME} is related to the electrical resistance R_E by

$$R_{ME} \propto \frac{B^2 l^2}{R_E}$$

The full derivation is given in Appendix 2 and it will be seen that series inductors on one side of the transducer will appear as parallel capacitors on the other and vice versa. This is illustrated by the difference in the circuits Figs. 2(a) and 2(b) and can be demonstrated by two practical effects.

(1) If the electrical impedance is noted at some low frequency and the cone is then touched, reducing its motion, the electrical impedance will be seen to *decrease* as a result of the *increase* in mechanical impedance.
(2) At resonance the cone velocity reaches maximum, indicating a minimum mechanical impedance characteristic of a *series LCR* circuit. The electrical impedance however will rise to a maximum characteristic of a *parallel LCR* circuit.

Regarded as an impedance-matching component the transducing element at a low frequency will have an optimum value for Bl. This should be such as to ensure that the cone maintains the condition of mass control down to the resonance of the system i.e. where the mass reactance of the cone equals the stiffness reactance of the suspension. This implies that for infinite baffle loading the Q of this resonance is unity. Often a Q of 0·5 is preferred since this gives the truly non-oscillatory condition and therefore secures the optimum transient performance. Also the mid and treble range efficiency is doubled. There is a 3 dB loss at the lowest working frequency but this is an acceptable sacrifice. The mechanical circuit Q is given by:

$$Q_M = \frac{2\pi f L_M}{R_M}$$

If R is mainly due to:

$$\frac{B^2 l^2}{10^9 R_E}$$

$$Q_M = \frac{2\pi f L_M R_E}{B^2 l^2} \cdot 10^9$$

$$Bl = \sqrt{\frac{2\pi f L_M R_E}{Q_M} \cdot 10^9}$$

Units are given in Appendix 1.

The design of the coil and magnet system should be determined by the above expression. The value chosen for $Q_M l$ will in fact be determined by the type of loading used but the aim will be the same, i.e. to maintain an overall system Q of between 0·5 and unity.

Observations (2). There is no advantage whatever to be had from a value of Bl greater than that above. Although mid- and high-frequency efficiency will be further increased this will be at the expense of the low frequency efficiency. The resulting "tilt" in the response may give a subjective impression of a better high-frequency transient response. In fact the transient performance at these frequencies is determined by quite different factors and is virtually unaffected by the value of Bl. In the case of all vented enclosure systems either increasing or decreasing Bl away from its optimum value will actually worsen the l.f. transient performance. This will be made clear in the next article.

In our Q calculations we used R_E to represent the electrical resistance of the voice coil and have ignored the output resistance of the amplifier which appears in series with it. The reason is that this is normally many times smaller than R_E. Some amplifier manufacturers make this resistance variable but if this *is* much less than R_E its precise value is of no consequence. If it is not much less than R_E then it is a very poor amplifier.

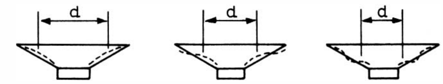

Fig. 6. Concentric flexure resulting in a reduction in effective cone diameter as frequency rises.

Loudspeaker cones

The foregoing analysis is restricted to the sloping part of the R_{MA} curve where it may be reasonably assumed that the cone will work as a substantially rigid piston. At higher frequencies however this is not so and the cone moves with different amplitudes and phase over different parts of its surface. It is this fact which enables a single cone loudspeaker to operate over a wide frequency range instead of falling at 12 dB/octave at above the R_{MA} knee as shown for the theoretical rigid piston in Fig. 5(a). Fig. 6 shows how a cone flexes concentrically at various high frequencies where the side of the cone becomes comparable to, or longer than, a wavelength. If we can assume that the incident wave is attenuated as it travels up the cone it will be seen that the effective cone diameter d reduces as frequency is raised.

Above the knee $R_{MA} \propto A \propto d^2$

Cone mass $L_M \propto A \propto d^2$

Radiated power $P_{MA} = v^2 R_{MA} \propto \frac{1}{d^4} \cdot d^2$

Therefore the reducing effective diameter tends to increase the radiated power as frequency rises thereby offsetting the condition in Fig. 5(a) and at the same time it broadens the polar response, this being a function of d (Appendix 1). It is readily possible by careful cone design to use this feature.

Another type of cone flexure is radial or bell-mode, shown in Fig. 7. This flexure can result in a very irregular frequency response

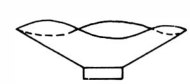

Fig. 7. Radial flexure or bell modes.

and transient ringing. This is particularly prevalent in straight sided cones but much less significant in sharply curved cones and can be virtually eliminated.

Observations (3). An interesting result occurs if we apply the above simple arithmetic proportionality argument to the situation below the R_{MA} knee.

Below the knee $R_{MA} \propto A^2 \propto d^4$

Cone mass $L_M \propto A \propto d^2$

Radiated power $P_{MA} = v^2 R_{MA} \propto \frac{1}{d^4} \cdot d^4$

This indicates that for a given cone material and a given applied force, the radiated power at low frequencies is independent of the cone diameter. However, there are two other considerations. If we use a smaller diameter cone we can for the same material reduce its thickness proportionally. Therefore:

$$\text{Cone Mass } L_M \propto d^3$$

Further we saw that to maintain the correct Q value

Bl (and therefore the force) $\propto \sqrt{L_M} \propto d^{1\cdot5}$

$\therefore$ radiated power $P_{MA} \propto \frac{d^{1\cdot5}}{d^6} \cdot d^4 \propto \frac{1}{\sqrt{d}}$

which indicates that the *smaller* the cone the *more efficient* it is at low frequencies.

The problem here is that to maintain the same radiated power one would expect that the cone displacement would increase in inverse proportion to the cone area. A few people imagining cone displacements of 2–3 in have cried "doppler distortion".

Now doppler distortion in this context, along with the Loch Ness monster, flying saucers and the Yeti, has provided a small band of devotees with an interest in life whilst the vast majority of people have been unaware of it. I am far too open minded to say these things do not exist. I can only say that after devoting a quarter of a century to the design and development of loudspeakers I have yet to encounter any significant distortion due to doppler effect. In any case with the far more efficient loading techniques practicable with small cones displacement need not normally exceed $\pm$0·125 in so the problem does not arise.

The single-cone loudspeaker

In order to achieve an extended coverage of the audio frequency range the cone needs to have a flared profile of hyperbolic form with the correct rate of flare. The effective reduction of area with increasing frequency can be arranged to compensate not only for the condition in Fig. 5(a) but also for the rising inductive reactance of the voice coil. The high-frequency limit of extension is approached when the reducing effective mass of the cone becomes comparable with the mass of the voice coil. There tends to be an efficiency maximum when these two are equal. In the case of the straight-sided cone the reduction of area is too rapid with the result that the output rises until again the effective cone mass equals the voice coil mass. The output then falls. This gives the peak usually around 5000 Hz, characteristic of these cones.

Polar distribution

This is very important. A level on-axis frequency response is quite useless if the off-axis response is falling. If the ear is to experience an adequate high-frequency performance this must be maintained off axis. Having said this however, we can add that for normal domestic applications a response that is maintained through a polar angle of about 60° is perfectly adequate. With the loudspeakers placed in their usual corner positions it would be unusual to find oneself listening outside this angle.

In this respect I would regard as excellent any loudspeaker that maintained a level treble response to 15 kHz or beyond at an angle of 30° off axis. I would also regard this as proving to be of much greater overall significance than the axial response since it gives a far better indication of h.f. power bandwidth.

Since the upper limit of the h.f. response is set by the voice-coil mass this must be kept as low as possible, consistent with reasonable efficiency. This compromise is usually resolved by the use of a very large magnet having a deep gap in which is immersed a short coil.

The cone diameter should be chosen so that the knee of the R_{MA} curve coincides with the effective area reduction.

The cone material poses some interesting problems. In general it needs to have a high stiffness-to-weight ratio and ideally a fairly high degree of internal friction. However, there is considerable likelihood that the normal mass, stiffness and internal friction properties of a material are vastly different when seen by a wave travelling in the material. Not only may these properties vary in a complex manner with frequency but also with amplitude. These problems started to interest me with the development of the titanium cone which provides much higher subjective definition than a corresponding aluminium cone. At the time of writing my article for the November 1966 issue of *Wireless World*† I was unable to find an adequate explanation for this in terms of the normally measured parameters.

The likely explanation, which has since emerged, is that after the incidence of any waveform the cone material must restore immediately to its original static position. The very soft material from which the aluminium cone was made had almost no elasticity so the cone was not fully restoring. This is a hysteresis effect and is particularly significant in materials where the internal friction is high compared with the material stiffness. Most mechanical damping materials exhibit a high degree of hysteresis.

Hysteretic distortion is a particularly insidious form of distortion upsetting frequency and transient response, and producing harmonic and intermodulation distortion. Usually the objective measurement of any one of these does not give any significant indication of hysteretic distortion but its combined effect on all these factors can make a complete mess of the subjective performance. Very often when faced with a resonant diaphragm it is tempting to apply some "gungy" damping material. This may certainly kill the ringing to the satisfaction of objective pulse tests. However, the resulting hysteretic distortion usually makes the subjective performance very much worse. Generally speaking hysteretic distortion is lower in metals than in papers, plastics or rubbers. These comments are applicable to all electromechanical transducers.

The cone surround is in every respect as important as the cone itself, in the effects it can have on sound quality. It has to

1. provide a highly flexible support for the cone edge;
2. provide an acoustically opaque seal to the enclosure;
3. completely absorb the incident concentric waves travelling up the cone at high frequencies; and
4. be completely non resonant.

A suitable surround material will have high density, high internal friction and be extremely soft and flexible. One of the best materials is highly plasticized p.v.c. sheeting but this is not a stable material. Various acrylic coatings on to polyurethane foam are being used with moderate success but application is difficult in production since a precise degree of impregnation is required.

There has just become available a new coating material which has precisely the right properties and is remarkably good for this application. Coated on to almost any speaker the improvement in treble smoothness is quite noticeable. The coating is very stable over very wide temperature ranges and completely waterproof. Further the quantity and method of application is not critical. A patent may be taken out on this application.

Observations (4). The single-cone high-quality loudspeaker has a great deal of objective argument in favour of it. Subjectively the approach can provide a sound quality that is outstanding, clean and well defined. Such loudspeakers can sometimes sound unkind on certain inputs and they have been criticized particularly by the American market as having inadequate power bandwidth. Further, the manufacturing processes are critical and unless close attention is paid to detail large variations between units and unreliability can result. I have often had the comment made to me, "Ted, the single cone loudspeakers are so very nearly right if only . . . etc."

According by request I have produced the design of a single cone loudspeaker which whilst broadly similar to previous units embraces a number of significant improvements. The high-frequency power response has been made smoother and more extended by redesign of cone and coil. The voice coil is both lighter and more efficient.

The radiated power at very low frequencies has also been very considerably increased by the use of a new type of loading. The power bandwidth is exceptionally wide. The overall performance has been balanced to provide a high standard of quality from first class inputs and an acceptable performance from indifferent inputs. The manufacturing processes have also been simplified with, it is hoped, an increase in repeatability and reliability. This loudspeaker is being manufactured by Audio Sound Techniques, of Leicester.

Cross-over systems

The alternative approach to securing a wide power bandwidth is to use separate loudspeaker units to cover discrete parts of the frequency range. A great deal of the loudspeaker design considerations already discussed apply also to the units used in crossover systems. It would seem to be a fundamental truism to say that it is a retrograde step to use two or more loudspeakers with their associated crossover matrices if one unit could do the job. Therefore, we must examine the areas in which this approach is justified.

The most significant advantage to be secured by crossover systems is that due to the fact that the bass unit cone can be large and massive, the low-frequency power-handling capacity may be extremely high. This is to some extent offset by the reduced efficiency which we have seen is characteristic of large heavy cones, but in many markets, particularly in the States where very high powered amplifiers are often used, the ability of a loudspeaker to handle these power levels without damage or noticeable distortion is of paramount importance.

The design of bass driver units follows exactly the same principles that we have already discussed. Their frequency range is normally limited to frequencies well below the knee of the R_{MA} curve, so that they should operate in the mass-control condition with the *Bl* factor determined as before. Since the cones are not required to flex they are constructed of either extremely thick hard paper or very often are formed solid from expanded polystyrene. This is sometimes coated with an aluminium skin to increase the rigidity but while it may do so as far as static forces are concerned it makes little difference to the rigidity as seen by oscillatory and transient forces. The adhesive used to stick the aluminium, however, may serve as a useful damping medium to the polystyrene which is highly resonant.

Mid-range units are usually more conventional cones since these are often required to straddle the knee of the R_{MA} curve and therefore need to flex in the way we have described.

For the high frequencies plastic-domed tweeters are popular. Again the dimensions and frequency range of these is such as to straddle the R_{MA} knee, and while such tweeters may be perfectly satisfactory on the slope of the R_{MA} curve they may experience difficulty with the range above the knee where flexure is required. If a cone or diaphragm is to flex, it must have the form of a transmission line where the force is applied at one end and the correct termination is applied at the other. In the case of a cone, the coil applies force in the centre and the surround provides the termination at the edge. The dome tweeter cannot meet these conditions, so any damping must be as a result of the internal friction of the material and since, in the case of plastics this is likely to be hysteretic, we may have a potentially unsatisfactory situation.

Both ionic and push-pull electrostatic tweeters are used in currently available crossover systems and these provide excellent high-frequency performances.

Crossover matrix design must be carried out experimentally. The use of formulae expressing the various values of inductance and capacitance in terms of crossover frequency and nominal impedance is unsatisfactory since the amplifier impedance is nearly zero and the impedance of moving-coil units is complex (Fig. 2b).

†E. J. Jordan, "Titanium Cone Loudspeaker," *Wireless World*, Nov. 1966.

The use of iron and ferrite cored inductors is undesirable. Any such core exhibits a high degree of hysteresis. The voltage developed across a ferrous cored inductor will only follow the applied voltage if this is derived from a zero impedance source. In the case of output transformers in valve amplifiers this condition can be met but with crossover systems it is not; the inductors will eventually have other impedances in series with them. The resulting hysteretic distortion can result in a complete loss of sound definition. Once again objective testing may not reveal the problem. A further point to watch is that at any significant power level a ferrous cone may be driven readily into saturation.

Observations (5). The development of a really good crossover system is not easy. In addition to the problems discussed above there is the difficulty of phase differences due to the physical spacing between units. Further, at the crossover frequency the voltage across one unit will be in phase advance the other in phase retard according to the matrix and also there is inevitably a step in the radiated power and/or polar response at the crossover frequency. These factors do not help the production of firm transient wavefronts.

I am also of the opinion that the majority of manufacturers of crossover type systems do not make full use of the potential advantages of the technique. The relative dimensions of the constituent units and the choice of crossover frequencies ought to be closely related. Instead they often appear to be chosen at random.

In spite of the many intrinsic problems good crossover systems can be designed and the problems can be overcome. A design could be provided, for example, which would provide mass controlled piston operation throughout its entire frequency range.

Conclusions

We have reached a stage in the art where the basic distortion forms can be objectively measured and dealt with. Given an engineer with some feeling for his work, loudspeakers can be produced which very adequately satisfy objective measurement and provide a very pleasant sound. One may be tempted to say that this is the end of the matter. From a purely commercial point of view it probably is and loudspeaker manufacturers may well wish to leave it at that. However, sooner or later someone is going to rock the boat. (Me for example).

Peter Walker caused a bit of a panic in the fifties with the full-range electrostatic loudspeaker. Every loudspeaker manufacturer frantically tried to catch him. However it was soon discovered that as a commercial proposition this approach was not on for the big boys. It was also discovered that you could not hit it with 35 W of sinewave at 30 Hz—which is a disadvantage in some markets. It is outside the scope of this article to discuss the design technology of the full-range electrostatic loudspeaker in any detail but it is very relevant at this stage to make some mention of its performance. The two particular features of the design are first that the diaphragms are driven equally all over their surfaces—thus tending to provide piston operation throughout the entire frequency range—and secondly, the diaphragms are driven under push-pull constant charge conditions.

Objectively the frequency response is smooth but not apparently better than that of many conventional systems. Non-linearity distortion is acceptably low throughout most of the range but below 100 Hz is higher than normally expected from better class units. As we have already indicated the power bandwidth leaves something to be desired particularly in the extreme bass. The transient response is excellent and the reproduction of square waves is superior to that of any other unit I have measured.

Subjectively the full-range e.s.l. can provide a standard of naturalness and realism not matched by dynamic systems. The high degree of definition and absence of colouration is quite outstanding. A point of particular interest is that these comments about the full range e.s.l. are pretty well universally shared which indicate that if a loudspeaker is good enough, people will agree about it.

The use of a moving-coil bass system and an electrostatic middle and top is the obvious thought to overcome the problem of bass power bandwidth but while this approach can provide a smooth pleasant performance, the definition of detail so apparent in the full-range e.s.l. is, in my experience severely reduced. It is worth noting that whilst the full-range e.s.l. uses a crossover system this is quite different from the type of matrix employed in conventional systems. The only effective reactive component is the leakage inductance of the signal transformer. Since the primary of this transformer is connected directly to the amplifier output and the transformer core is of extremely high-quality hysteretic distortion is minimized.

It seems to me now that the aim of future development should be to achieve the definition standard set by the full-range e.s.l. coupled to the wide-power bandwidth which we have come to associate with American loudspeakers. I, personally feel that this situation is most likely to be resolved for the time being by further development of the single-cone approach coupled to improved loading techniques. I believe I can also see the next major step in loudspeaker development—but that is a story for a later date.

APPENDIX 1

Radiation impedance

Radiation impedance is given by the Bessell series.

$$Z_{MA} = \rho c \pi r^2 \left\{ \left[\frac{(2kr)^2}{2.4} - \frac{(2kr)^4}{2.4^2.6} + \frac{(2kr)^6}{2.4^2.6^2.8} \cdots \text{etc.} \right] + j\frac{4}{\pi}\left[\frac{2kr}{3} - \frac{(2kr)^3}{3^2.5} + \frac{(2kr)^5}{3^2.5^2.7} \cdots \text{etc.} \right] \right\}$$

where

$$k = \frac{2\pi f}{c} \quad \text{and} \quad c = 3{\cdot}44 \times 10^4 \text{ cm/sec}$$

From this the following approximate equations can be derived. Below the knee of the curve where $kr \ll 2$

$$R_{MA} \approx \frac{\rho c k^2}{2\pi} (\pi r^2)^2$$

$$X_{MA} \approx \frac{8}{3} \rho c k r^3 \text{ g}$$

where

$$\rho = 1{\cdot}21 \times 10^{-3} \text{ g/cc}$$

Above the "knee" of the curve where $kr \gg 2$

$$R_{MA} \approx \rho c \pi r^2 \text{ mech. ohms (g/cm/sec)}$$

$$X_{MA} \approx \frac{2\rho c r}{k} \text{ g}$$

All the foregoing expressions are for *mechanical* impedance Z_{MA} due to the air load. If the expressions are divided by $(\pi r^2)^2$, $(=A^2)$ we obtain the acoustical impedance Z_A

Directivity

The ratio of pressure p_θ at an angle θ' degrees off axis to the pressure p_0 at the same radial distance on axis is given by

$$\frac{p_\theta}{p_0} = 1 - \frac{kr \sin\theta}{8}.$$

APPENDIX 2

Relationship between mechanical and electrical impedances.

$$e_b = \frac{Blv}{10^8} \text{ volts}$$

$$v = \frac{\text{Force}}{Z_M} = \frac{Bli}{10 Z_M} \text{ cm/sec}$$

$$\therefore e_b = \frac{B^2 l^2 i}{10^9 Z_M}$$

Electrical impedance Z_{EM} due to mechanical impedance Z_M is given by

$$Z_{EM} = \frac{e_b}{i} = \frac{B^2 l^2}{10^9 Z_M} = \frac{1}{R_M + j\left(\omega L_M - \frac{1}{\omega C_M}\right)} \cdot \frac{B^2 l^2}{10^9}$$

From this the following relationships can be derived.

$$R_{EM} = \frac{1}{R_M} \cdot \frac{B^2 l^2}{10^9} \text{ ohms}$$

$$\omega C_{EM} = \omega L_M \cdot \frac{10^9}{B^2 l^2} \text{ mhos}$$

$$\omega L_M = \omega C_M \cdot \frac{B^2 l^2}{10^9} \text{ ohms.}$$

Also

$$Z_{ME} = \frac{1}{Z_E} \cdot \frac{B^2 l^2}{10^9} \text{ mech. ohms (g/cm/sec)}$$

$$R_{ME} = \frac{1}{R_E} \cdot \frac{B^2 l^2}{10^9} \text{ mech. ohms (g/cm/sec)}$$

$$\omega C_{ME} = \omega L_E \cdot \frac{10^9}{B^2 l^2} \text{ mech. mhos (cm/dyne)}$$

$$\omega L_{ME} = \omega C_E \cdot \frac{B^2 l^2}{10^9} \text{ mech. ohms (g/cm/sec).}$$