

## Grid-Volts/Mutual-Conductance Characteristic Applied to Detectors and Modulators

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**I**N the June, 1951, issue of this journal I discussed the way in which the single valve curve, mutual conductance as a function of grid voltage, could be used to decide which of several similar valve types should be used in an amplifier. The article contained no mention of the effect of local feedback, because it was based on the premise that overall negative feedback would be used in any amplifier that any reader is likely to design, and it did not consider any of the other applications of valves, of which the modulator is the most important. These topics will be discussed in this article.

It may be convenient to recapitulate some of the results derived in the first article, and correct some of the formulæ. We took in equation (1) the anode current  $I_a$  as a function of the steady current with no signal applied  $I_o$ , the grid voltage  $e_g$ , and parameters of the valve A, B, C.

$$I_a = I_o + Ae_g + Be_g^2 + Ce_g^3 + \dots \quad (1)$$

and

$$\frac{dI_a}{de_g} = A + 2Be_g + 3Ce_g^2 + \dots \quad (2)$$

Here, then, A is the mutual conductance of the valve, and at any voltage  $e_g$  we have

$$g = g_o + 2Be_g + 3Ce_g^2 + \dots \quad (3)$$

Differentiating again,  $\frac{dg}{de_g} = 2B + 6Ce_g + \dots$

Referring now to Figs. 1 and 2 in the earlier paper  $2B = g/2e$  and thus  $B = g/4e$ .

The 2nd-harmonic distortion is thus  $(g/8g_o)$  100%. This was incorrectly given as  $(g/4g_o)$ . 100 per cent. in the previous article. In consequence, when third harmonic distortion is considered we will have more than second if  $\delta$ , the deviation from linearity (Fig. 2), is greater than  $g/24$ . Fig. 1 shows a  $g_m - e_g$  characteristic which is a straight line in the working region, so that C and higher terms of  $e_g$  vanish: the  $I_a - e_g$  curve is then a parabola.

The characteristic shown in Fig. 2 is quite common, especially among the small pentodes. This, in addition to producing second harmonic, which can be calculated by the expression above, also produces third harmonic, at a level  $3\delta/g_o$ . 100 per cent of the fundamental. For the curve shown this harmonic is phased so that it increases the peaks of the wave, while if the curve is concave downwards the third harmonic is of the right phase to make the peak value less and thus has a squaring effect.

Let us now consider what happens when we leave the cathode bias resistor of the valve unbypassed, so that there is a certain amount of negative feedback in the stage. The most useful characteristic will be

a graph of mutual conductance against grid-earth voltage, which is what we should measure if the cathode resistor were built into the bulb. We do in fact, measure just this quantity on old valves which have built up a barrier layer at the cathode surface.

The effect of the bias resistor is to make  $e_g = e_o - I_a R_k$  and we can rearrange this equation as:

$$(e_o - e_g) = R_k I_a$$

Differentiating, we have

$$\frac{de_o}{dI_a} - \frac{de_g}{dI_a} = R_k \quad \text{or} \quad \frac{de_o}{dI_a} = \frac{de_g}{dI_a} + R_k$$

Now  $\frac{de_o}{dI_a}$  is just the reciprocal of the mutual conductance with the cathode resistor in circuit,  $g_k$ ,

so that  $\frac{1}{g_k} = \frac{1}{g_m} + R_k$

For any particular grid-cathode voltage, therefore, we obtain the effective mutual conductance by adding the cathode resistance to the reciprocal of the valve mutual conductance. In Fig. 3 this has been done, using a cathode resistance of  $1/g_o$  where  $g_o$  is the mutual conductance at the working point. This is a fairly common value, and corresponds to 6 db of feedback. It will immediately be observed that the value of  $g$  is

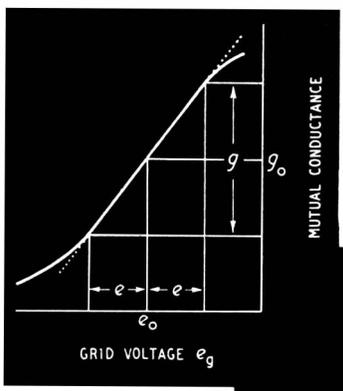


Fig. 1. The simplest type of valve  $g_m - e_g$  characteristic has a relatively long linear section.

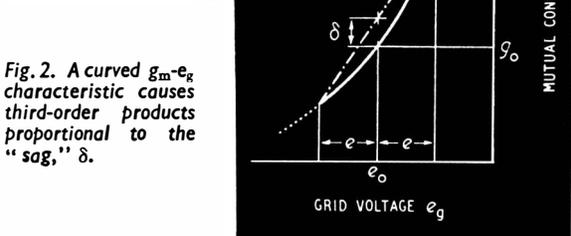


Fig. 2. A curved  $g_m - e_g$  characteristic causes third-order products proportional to the "sag,"  $\delta$ .

halved for small grid excursions, so that, as we expect, the distortion is halved, but that if the valve is driven at all hard there is a substantial third harmonic component present. For the example shown, indeed, the condition  $\delta > g/24$  is rapidly reached, so that there is more third harmonic than second. This third harmonic component is of the phase which tends to square the waveform, a result which will surprise no one who has examined the overload effects in an amplifier with a large amount of negative feedback.

It will not be without interest to notice how, in Fig. 4, a third-order characteristic has been straightened out to almost the straight line we associate with a second-order characteristic. A very careful examination would show the appearance here of higher-order wobbles in the curve, but the accuracy with which we can determine valve characteristics does not justify too critical an examination. The general results can be summarized by saying that a cathode resistor tends to bow the curve upwards, with particularly beneficial effects on characteristics like that of Fig. 2, and at the same time introduces harmonics of higher order.

For some special applications it is desirable to produce a very linear amplifier without using overall negative feedback. It will be clear that by adjusting the stage gains by choice of anode load, and the

stage distortion by choice of cathode resistor, a fair amount of distortion balancing can be achieved, since alternate stages tend to balance the second harmonic, and "concave upwards" stages tend to balance the third harmonic from concave downwards stages.

The appearance of the third harmonic when we add negative feedback to the parabolic characteristic, or straight  $g_m - e_c$  curve, which alone produces only second harmonic, is easily explained. A single frequency input between grid and cathode produces a second harmonic term at the cathode, owing to the current in the cathode resistor. We now have between grid and earth both fundamental and second harmonic: conversely, with a single frequency input between grid and earth, both fundamental and second harmonic appear between grid and cathode. The valve is not linear, and acts as a modulator, producing terms  $2f \pm f$  from the terms  $f$  and  $2f$  which are present between grid and cathode, and there is your third harmonic.

We must now go on to discuss the choice of a valve for use as a modulator or a detector. If we take the parabolic pentode, with a straight line  $g_m - e_c$  characteristic, we have  $I_a = I_o + g_m(e \sin \omega t) + \frac{B e^2}{2} (1 - \cos 2\omega t)$  where  $2B = dg_m/de_c$ . This equation is given at the top of p. 222 of the June 1951 issue. The direct-current component produced in the anode circuit by the application of the signal is thus

$$I_r = \frac{1}{2} B e^2.$$

It is not difficult to show that if signals  $e_1 \sin \omega_1 t, e_2 \sin \omega_2 t \dots e_n \sin \omega_n t$  are applied simultaneously to the grid, the anode current increment is

$$I_{g,r} = \frac{1}{2} B (e_1^2 + e_2^2 + \dots e_n^2)$$

provided that  $e_1 + e_2 \dots e_n$  does not exceed the voltage for which the linear relationship between  $g_m$  and  $e_c$  holds. The valve thus provides an anode current change proportional to the square of the r.m.s. input, and the anode current meter can be calibrated directly in true r.m.s. volts. The maximum current which can be obtained is  $\frac{1}{2} g_e e_c$  and this, for a perfect valve having a cut-off at  $e_c$  volts and an average mutual conductance of  $g_{av}$ , will be  $1/16 g_{av} e_c$ . The standing current, in the absence of any signal, will be  $\frac{1}{2} g_e e_c$ , so that we cannot have more than 25 per cent rise in anode current.

For a detector, the criterion is clearly a linear  $g_m - e_c$  characteristic, and a maximum area under it. If the characteristic is not fully linear, we must compare the area under the linear portion with the area under the whole curve. Fig. 5 shows the two areas to be compared. The waste area, resulting from the tail on the characteristic, increases the standing current from zero is to be used. The larger this standing current is for a given value of  $I$ , the more difficult it will be to keep the system in the balanced condition. To compare two valves, therefore, we should use the ratio of these areas as an efficiency figure, and choose the valve which gives the best ratio.

An examination of the equation given above shows that the second harmonic current is equal to the rectified current, so that in a frequency doubler the change of anode current when drive is applied is a direct measure of the second harmonic term. For both these cases the use of a cathode resistor may be of assistance if a third harmonic component of the concave upwards type is present.

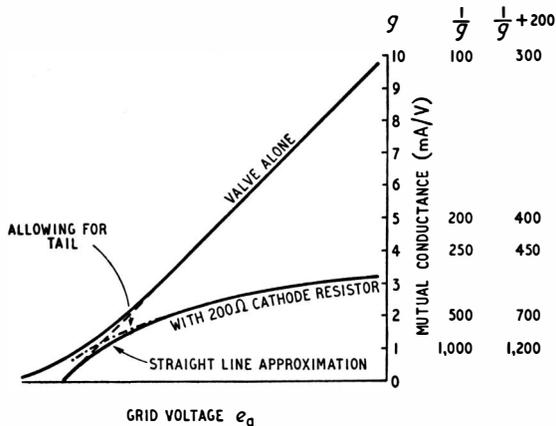


Fig. 3. The use of an unbypassed cathode resistor makes the effective mutual conductance more constant, but adds higher-order terms to the valve characteristic.

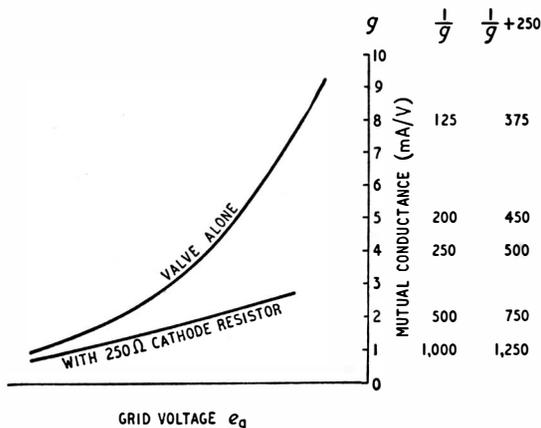
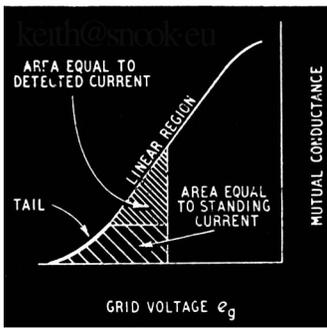


Fig. 4. A curved  $g_m - e_g$  characteristic may be straightened by a cathode resistor. This is important in modulator design.

Fig. 5. The efficiency of a square-law detector depends on the length of the tail of the characteristic.



When the valve is to be used as a modulator, we shall apply a composite signal  $e_1 \sin \omega_1 t + e_2 \sin \omega_2 t$  to the grid, giving  $I_a = I_o + g_m(e_1 \sin \omega_1 t + e_2 \sin \omega_2 t) + B(e_1 \sin \omega_1 t + e_2 \sin \omega_2 t)^2 = I_o + g_m(e_1 \sin \omega_1 t + e_2 \sin \omega_2 t) + B(e_1^2 \sin^2 \omega_1 t + e_2^2 \sin^2 \omega_2 t + 2Be_1e_2 \sin \omega_1 t \sin \omega_2 t)$ .

The first two terms in this expression can be neglected, as they are the standing current and the separately amplified signals. The third term reduces to

$$\frac{B}{2}(e_1^2 + e_2^2 - [e_1^2 \cos 2\omega_1 t + e_2^2 \cos 2\omega_2 t])$$

a direct current term and a term containing the two second harmonics. The fourth term is the important modulation term:  $2Be_1e_2 \sin \omega_1 t \sin \omega_2 t$

Each modulation term is therefore of amplitude  $Be_1e_2$ . If  $e_1$  is very much larger than  $e_2$  we can take

$$e_1 = e \text{ giving } I_{as} = \frac{g}{4} e_2 \text{ where } I_{as} \text{ is the anode current}$$

component of one sideband. The conversion conductance is  $g/4$  which is certainly less than  $g_o/8$ . The term in the anode current corresponding to the signal  $e_2$  is equal to  $g_o e_2$  so that the filter which separates sideband from signal must deal with a signal  $4g_o/g$  above the sideband. We must make  $g/g_o$  as large as possible. The anode current also contains the carrier term,  $g_e e_1 = g_o e$  so that here we must filter out a term  $4g_o e/g$  times as large as the wanted sideband.

Since  $(e_1 + e_2)$  must not exceed  $e_s$ , we can see how to get maximum output. The sideband output depends on the product  $e_1 e_2$ , so that a very simple manipulation reveals that maximum sideband output is obtained for  $e_1 = e_2 = e/2$ . We then have  $I_{as} = Be^2/4 = ge/16$ .

This is the maximum output which we can get from a mixer in a device like a beat-frequency oscillator, which has ample supplies of both oscillator voltages available. Each of the sources will produce an anode current component equal to  $g_o e/2$  and a second harmonic term of  $ge/16$ . This enables us to estimate the filtering which must be provided. When operating in frequency generating equipment it is often desirable to keep the ratio of sideband to a fundamental maximum. In this case we want  $g/g_o$  to be as large as possible. Otherwise, for maximum sideband output it is the product  $ge$ , the area under the straight part of the curve, which must be large.

A radio-frequency amplifier must not introduce cross-modulation: the modulator analysis is therefore applicable, and since we are only interested in third-order products, or higher odd terms, we see that the coefficient  $\delta$  is the determining one. The second-order terms  $f_1 \pm f_2$  are not important, because they fall far away from the frequency range in use. In

theory, then, we need not worry about the value of  $g$ , provided  $\delta/g_o$  is very small. In practice it is advantageous to choose a valve with a small value of  $g/g_o$ , because any second order terms which reach a later stage will then mix under second-order conditions to provide third-order terms.

I do not propose to consider what happens modulators with higher-order characteristics, because for almost every purpose the choice lies between using the simple parabolic characteristic, even if a cathode resistor must be added to smooth out a third-order concavity, or using a discontinuous characteristic to provide a high-order term.

Analysis of triode performance is less easily carried out in this simple way, unless the anode load is very small. If the anode load is large, a separate  $g_m - e_o$  characteristic must be constructed for each possible anode load. An example is given in Fig. 6, in which the mutual conductance for an anode load of 12,500 ohms is obtained by taking the change of anode current for each 1-volt step. Travelling down the load line from  $E_o = 0$  to  $E_o = -1$  gives a change of 2.3 mA in the anode current, which is plotted as a mutual conductance of 2.3 mA/volt at  $E_g = -0.5$ . This is actually a synthetic example, which is why the excellent linearity of the  $g_m - e_o$  characteristic is observed. Any triode can be studied quite critically by plotting a set of these  $g_m - e_o$  characteristics with different values of anode load. The effect of cathode-resistor feedback can be analysed either by plotting the modified plate characteristics or, perhaps more easily, by operating on the  $g_m - e_o$  characteristics.

This survey of the use of a single valve curve is by no means comprehensive: to make it so we should need to consider high-order terms and carry out much more complete mathematical analyses. And then we should be faced by the problem of obtaining sufficiently accurate experimental data to insert in our formulae. The only way in which such data could be obtained would be by measurement of the high-order products in the anode current, which we could then use, in the long run, to calculate these products. All we need in practice is a knowledge of the slope and sag,  $g$  and  $\delta$ , and a knowledge of how these are affected by cathode-resistor feedback. We are then in a position to understand what is happening in any particular case.

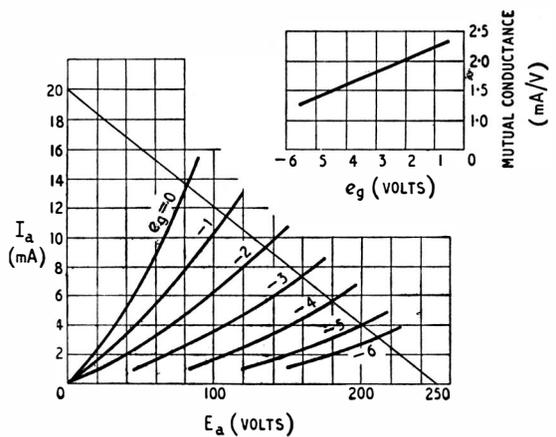


Fig. 6. For a triode it is necessary to fix the load line before plotting the  $g_m - e_g$  characteristic. This typical example shows a linear  $g_m - e_g$  characteristic for  $R_L = 12,500$  ohms.